Term Information

Effective Term	Autumn 2023

General Information

Course Bulletin Listing/Subject Area	Mathematics
Fiscal Unit/Academic Org	Mathematics - D0671
College/Academic Group	Arts and Sciences
Level/Career	Undergraduate
Course Number/Catalog	4345
Course Title	Formal Proof
Transcript Abbreviation	Formal Proof
Course Description	This course provides an introduction to formal proof in mathematics. It is designed for math majors who seek a comprehensive under- standing of how to use proof assistants and how to encode a mathematical proof in such a way that one can formally verify its correctness.
Semester Credit Hours/Units	Fixed: 3

Offering Information

Length Of Course	14 Week, 12 Week, 8 Week, 7 Week, 6 Week
Flexibly Scheduled Course	Never
Does any section of this course have a distance education component?	No
Grading Basis	Letter Grade
Repeatable	No
Course Components	Lecture
Grade Roster Component	Lecture
Credit Available by Exam	No
Admission Condition Course	No
Off Campus	Never
Campus of Offering	Columbus

Prerequisites and Exclusions

Prerequisites/Corequisites	A grade of C- or better in 3345 or permission of department.
Exclusions	
Electronically Enforced	Yes

Cross-Listings

Cross-Listings	

Subject/CIP Code

Subject/CIP Code	27.0101
Subsidy Level	Baccalaureate Course
Intended Rank	Junior, Senior

Requirement/Elective Designation

The course is an elective (for this or other units) or is a service course for other units

Course Details

Course goals or learning objectives/outcomes

- The student will be able to use a proof assistant to write formal proofs of basic mathematical theorems, using a variety of techniques such as induction, contradiction, case analysis;
- The student will be able to read and understand existing formal proofs and identify techniques used;
- The student will be able to write clear and concise mathematical prose commenting on formal proofs;
- The student will collaborate effectively with other students in writing and reviewing formal proofs, and provide constructive feedback on their peer's work;
- The student will be able to explain the correspondence between proofs and programs;
- The student will be able to make use of a library of previously proved results (like mathlib) to make progress on problems in various areas of mathematics such as algebra, analysis, geometry, and combinatorics;
- The student will understand fundamental concepts of dependent type theory such as introduction and elimination rules, and relate those rules to the writing of formal proofs.

Content Topic List

- Natural numbers and induction; Order and divisibility
- Logic and propositions; Proof terms and tactics
- Examples: inequalities and sequences
- Functions
- Set theory
- Equivalence relations and quotients
- Induction and recursion
- Examples: number theory, point-set topology, metric spaces, abstract algebra

Sought Concurrence

No

Attachments

- 4345syllabus.pdf: Syllabus-old

  (Syllabus. Owner: Husen,William J)
- Curriculum_map_math_03272023.docx: Curriculum map

  (Other Supporting Documentation. Owner: Husen,William J)
- 4354_syllabus_revised.pdf: Syllabus-revised

  (Syllabus. Owner: Husen,William J)

Comments

- Updated syllabus attached addressing approval contingencies. *(by Husen,William J on 05/11/2023 11:37 AM)*
- Please see feedback email sent to department 04-28-2023 RLS *(by Steele,Rachel Lea on 04/28/2023 11:32 PM)*

Workflow Information

Status	User(s)	Date/Time	Step
Submitted	Husen,William J	03/27/2023 01:15 PM	Submitted for Approval
Approved	Husen,William J	03/27/2023 01:23 PM	Unit Approval
Approved	Vankeerbergen,Bernadette Chantal	04/05/2023 12:55 PM	College Approval
Revision Requested	Steele,Rachel Lea	04/28/2023 11:32 PM	ASCCAO Approval
Submitted	Husen,William J	05/11/2023 11:37 AM	Submitted for Approval
Approved	Husen,William J	05/11/2023 11:37 AM	Unit Approval
Approved	Vankeerbergen,Bernadette Chantal	05/27/2023 06:48 PM	College Approval
Pending Approval	Jenkins,Mary Ellen Bigler Hanlin,Deborah Kay Hilty,Michael Vankeerbergen,Bernadette Chantal Steele,Rachel Lea	05/27/2023 06:48 PM	ASCCAO Approval

Formal Proof: Sample Syllabus

May 6, 2023

Course Title: Formal Proof
Credit Hours: 3
Instruction format: Lecture

Textbook: *Mathematics in Lean* by Jeremy Avigad, Kevin Buzzard, Robert Lewis, Patrick Massot.

This textbook is driven by examples, and this course similarly involves examples drawn from across the mathematical landscape.
Prerequisites/Corequisites:: A grade of $C-$ or above in Math 3345.

Description. This course provides an introduction to formal proof in mathematics. It is designed for math majors who seek a comprehensive understanding of how to use proof assistants and how to encode a mathematical proof in such a way that one can formally verify its correctness. Throughout the course, students will engage in weekly homework assignments, a midterm project, and a final project. By the end of this course, students will be able to take their prior mathematical learning and communicate it in a format suitable for machine verification.

Course goals or learning objectives/outcomes. By the end of the course,

- the student will be able to use a proof assistant to write formal proofs of basic mathematical theorems using a variety of techniques such as induction, contradiction, case analysis;

- the student will be able to read and understand existing formal proofs and identify techniques used;
- the student will be able to write clear and concise mathematical prose commenting on formal proofs;
- the student will collaborate effectively with other students in writing and reviewing formal proofs and provide constructive feedback on their peers' work;
- the student will be able to explain the correspondence between proofs and programs;
- the student will be able to make use of a library of previously proved results (like `mathlib`) to make progress on problems in various areas of mathematics such as algebra, analysis, geometry, and combinatorics;
- the student will understand fundamental concepts of dependent type theory such as introduction and elimination rules, and relate those rules to the writing of formal proofs.

Planned curriculum

Course content is drawn from the *Mathematics in Lean* textbook, with weekly worksheets reinforcing the content from the textbook. About half the course is "examples" meaning that some basic definitions from a given area of mathematics are formalized and students are challenged to prove fundamental results.

Week 1 Natural numbers and induction

> **Homework 1** using `https://www.ma.imperial.ac.uk/~buzzard/xena/natural_number_game/` which, being web-hosted, does not require local installation, thereby ensuring students can get to work while they figure out how to install Lean locally on their own machines
>
> **Reading** from §2.2–2.3 of the textbook

Week 2 Order and divisibility

Homework 2 ensuring Lean is running locally on their machines

Reading from §2.4–2.5 of the textbook

Week 3 Logic and propositions

Homework 3 translating material about propositions from Math 3345 into Lean

Reading from §3.1–3.3 of the textbook

Week 4 Proof terms and tactics

Homework 4 introducing new proof tactics

Reading from §3.4–3.5 of the textbook

Week 5 Examples: sequences

Homework 5 proving "calculus 2" results about convergence

Reading from §3.6 of the textbook

Week 6 Examples: inequalities

Homework 6 providing some suggestions for the midterm project

Reading and reviewing §2.1 of the textbook

Week 7 Set theory

Homework 7 translating some set theory from Math 3345 into Lean

Reading from §4.1 of the textbook

Week 8 Functions

Homework 8 centering around injectivity and surjectivity

Reading from §4.2 of the textbook

Week 9 Equivalence relations and quotients

Midterm Project due

Homework 9 proving results about modular arithmetic

Week 10 Induction and recursion

Project Feedback due

Homework 10 reviewing deeper results from the "natural number game" in Week 1

Week 11 Examples: number theory

Homework 11 culminating in the proof of the infinitude of primes

Reading from §5.1–5.3 of the textbook

Week 12 Examples: metric spaces

Homework 12 culminating in showing the composition of continuous functions is continuous

Reading from §7.2 of the textbook

Week 13 Examples: point-set topology

Homework 13 introducing basic definitions from topology

Reading from §7.3 of the textbook

Week 14 Examples: abstract algebra

Reading from §6.1–6.3 of the textbook

Assessment

There are 500 points possible in this course; earning an A or B or C or D requires earning 450 or 400 or 350 or 300 points, respectively. These points are broken down as follows.

Homework (260 points; 20 points each). There will be thirteen homework assignments posted on the course website. You will receive a portion of 20 points for each of these assignments based on your performance. Your work should be well-organized and should provide well-commented code and clear written arguments where appropriate. **Although collaboration is not required, your submitted homework must list those with whom you have collaborated if you choose to work in a group; be sure to explain your contributions to the group's effort.** Homework is generally due on Fridays.

Midterm and final project (240 points; 120 points each). Each project involves writing a formal proof for a result you have learned in a previous math course. Projects must include both code and a write-up (of a few pages) explaining the approach you have taken. Midterm projects will also be made accessible for peer review, with each student expected to offer constructive feedback on at least one other midterm project. Scores will be based on the quality of the project's code, the clarity of the written explanation, and, for the midterm project, the quality of the feedback offered on another project. Midterm projects are due at the end of Week 9 and feedback is due at the end of Week 10.

You are encouraged, but not required, to use LaTeX for your written explanations and when providing feedback to your fellow students. However, the primary focus of the homework assignments and both projects is coding. The language used for these coding tasks is "Lean," a freely available programming language specifically designed for formal verification and interactive theorem proving. We recommend using Lean via Visual Studio Code on a laptop running Windows, macOS, or Linux. The experience is largely the same on all three platforms. (You may also access Lean via emacs if you are already familiar with emacs.) To assist with onboarding, the first week of this course is built around a web-hosted version of Lean, so you will not have to install Lean on your own machine until Week 2.

Academic integrity policy

Your written assignments must be your own work, but you should discuss problems with other students in this course and seek out additional resources and readings. The write-up must be entirely your own. **Your submitted homework must list those with whom you have collaborated.** Limit your collaboration discussing general strategies and concepts. Directly sharing or copying large amounts of code, or using code snippets from outside resources without proper citation, is prohibited. If you are unsure about a particular situation, please ask the instructor.

Academic Misconduct Statement

It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335–5–48.7). For additional information, see the Code of Student Conduct at `http://studentlife.osu.edu/csc/`

Statement on title IX

Title IX makes it clear that violence and harassment based on sex and gender are Civil Rights offenses subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories (e.g., race). If you or someone you know has been sexually harassed or assaulted, you may find the appropriate resources at `http://titleix.osu.edu` or by contacting the Ohio State Title IX Coordinator, Kellie Brennan, at `titleix@osu.edu`.

Disability Services Statement

The University strives to make all learning experiences as accessible as possible. In light of the current pandemic, students seeking to request COVID-related accommodations may do so through the university's request process, managed by Student Life Disability Services. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After regis-

tration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; slds.osu.edu; 098 Baker Hall, 113 W. 12th Avenue.

Religous accomodations

Our inclusive environment allows for religious expression. Students requesting accommodations based on faith, religious or a spiritual belief system in regard to examinations, other academic requirements or absences, are required to provide the instructor with written notice of specific dates for which the student requests alternative accommodations at the earliest possible date. For more information about religious accommodations at Ohio State, visit odi.osu.edu/religious-accommodations.

Your mental health

As a student you may experience a range of issues that can cause barriers to learning, such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished academic performance or reduce a student's ability to participate in daily activities. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you or someone you know are suffering from any of the aforementioned conditions, you can learn more about the broad range of confidential mental health services available on campus via the Office of Student Life's Counseling and Consultation Service (CCS) by visiting ccs.osu.edu or calling 614-292-5766. CCS is located on the 4th Floor of the Younkin Success Center and 10th Floor of Lincoln Tower. You can reach an on call counselor when CCS is closed at 614-292-5766 and 24 hour emergency help is also available 24/7 by dialing 988 to reach the Suicide and Crisis Lifeline.

Math - BS/BA Curriculum Map					
Goal 1	**Learn conceptual frameworks needed to study higher mathematics, including an introduction to mathematical reasoning and an understanding of how to read and write proofs.**				
Goal 2	**Aquire basic mastery of core areas of mathematics including calculus, analysis and algebra.**				
Goal 3	**Develop powerful mathematical problem solving skills.**				
Goal 4	**Learn to communicate mathematical understanding effectively.**				
Goal 5	**Become proficient in chosen tracks within the major.**				
Course	**Goal 1**	**Goal 2**	**Goal 3**	**Goal 4**	**Goal 5**
AcctMIS 2000			Beginning		Intermediate
Biochem 4511					Advanced
Biology 1113			Beginning		Intermediate
Biology 1114			Beginning		Intermediate
Biology 3401					Intermediate
BusFin 3120			Intermediate	Intermediate	Advanced
BusFin 3220			Intermediate	Intermediate	Advanced
Chem 1210			Beginning		Intermediate
Chem 1220			Beginning		Intermediate
Chem 2210					Advanced
Chem 2510					Advanced
Chem 4300					Advanced
Chem 4310					Advanced
CSE 1222			Beginning		Intermediate
CSE 1223			Beginning		Intermediate
CSE 2221			Beginning	Beginning	
CSE 2111			Beginning		Intermediate
Econ 2001.01			Beginning		Intermediate
Econ 2002.01			Beginning		Intermediate
EEOB 3310					Advanced
EEOB 3420					Advanced
EEOB 4520					Advanced
Math 1151	Beginning	Beginning	Beginning		
Math 1152	Beginning	Beginning	Beginning		
Math 1181H	Intermediate	Intermediate	Beginning		
Math 1295				Intermediate	Beginning
Math 2153	Intermediate	Intermediate	Beginning		

Math 2182H	Intermediate	Intermediate	Beginning		
Math 2255	Beginning	Intermediate	Intermediate	Beginning	
Math 2568	Beginning	Beginning	Beginning		Beginning
Math 2568H	Intermediate	Beginning	Intermediate	Beginning	Beginning
Math 3345	Advanced	Advanced	Intermediate	Intermediate	Intermediate
Math 3345H	Advanced	Advanced	Intermediate	Intermediate	Intermediate
Math 3350				Intermediate	Beginning
Math 3589			Intermediate	Intermediate	Advanced
Math 3607			Intermediate	Intermediate	Advanced
Math 3618			Intermediate	Advanced	Advanced
Math 4181H	Advanced	Advanced	Advanced	Advanced	Advanced
Math 4182H	Advanced	Advanced	Advanced	Advanced	Advanced
Math 4345	**Advanced**	**Advanced**	**Advanced**	**Intermediate**	**Advanced**
Math 4350			Intermediate	Advanced	Advanced
Math 4504	Advanced	Intermediate	Intermediate	Advanced	Advanced
Math 4507	Advanced	Intermediate	Intermediate	Advanced	Advanced
Math 4512	Intermediate		Intermediate	Intermediate	Intermediate
Math 4530	Intermediate	Beginning	Intermediate	Intermediate	Intermediate
Math 4547	Advanced	Advanced	Intermediate	Advanced	Beginning
Math 4548	Advanced	Advanced	Intermediate	Advanced	Beginning
Math 4551	Intermediate	Intermediate	Intermediate	Intermediate	Intermediate
Math 4552	Intermediate	Intermediate	Intermediate	Intermediate	Intermediate
Math 4556			Intermediate	Advanced	Advanced
Math 4557	Intermediate		Intermediate	Intermediate	Intermediate
Math 4570	Intermediate	Intermediate	Advanced	Intermediate	Intermediate
Math 4573	Advanced	Intermediate	Intermediate	Intermediate	Intermediate
Math 4575	Intermediate	Intermediate	Intermediate	Intermediate	Intermediate
Math 4578	Intermediate	Intermediate	Intermediate	Intermediate	Advanced
Math 4580	Advanced	Advanced	Intermediate	Advanced	Beginning
Math 4581	Advanced	Advanced	Intermediate	Advanced	Beginning
Math 5101	Beginning	Advanced	Intermediate		Intermediate
Math 5102	Beginning	Advanced	Intermediate		Intermediate
Math 5421	Beginning	Beginning	Intermediate	Beginning	Advanced
Math 5451	Beginning	Beginning	Intermediate	Beginning	Advanced

Math 5520H	Advanced	Advanced	Advanced	Advanced	Intermediate
Math 5522H	Advanced	Advanced	Advanced	Advanced	Intermediate
Math 5529H	Advanced	Advanced	Advanced	Advanced	Intermediate
Math 5530H	Advanced	Advanced	Advanced	Advanced	Intermediate
Math 5540H	Advanced	Advanced	Advanced	Advanced	Advanced
Math 5540H	Advanced	Advanced	Advanced	Intermediate	Beginning
Math 5571	**Advanced**	**Advanced**	**Advanced**	**Intermediate**	**Intermediate**
Math 5576H	Advanced	Advanced	Advanced	Advanced	Advanced
Math 5590H	Advanced	Advanced	Advanced	Advanced	Advanced
Math 5591H	Advanced	Advanced	Advanced	Advanced	Advanced
Math 5632			Intermediate	Advanced	Advanced
Math 5635			Intermediate	Advanced	Advanced
Math 5636			Intermediate	Advanced	Advanced
Math 5637			Intermediate	Advanced	Advanced
Math 5660					Intermediate
Math 5756			Beginning	Intermediate	Intermediate
Math 5757			Beginning	Intermediate	Intermediate
MolGen 4500					Advanced
MolGen 5601					Advanced
Physics 1250			Beginning		Intermediate
Physics 1251			Beginning		Intermediate
Physics 2300					Advanced
Physics 2301					Advanced
Stat 4201	Intermediate	Beginning	Intermediate	Intermediate	Intermediate
Stat 4202	Intermediate		Intermediate		Intermediate